9.5

Series

Summation Notation

In **summation notation,** the sum of the terms of the sequence $\{a_1, a_2, a_3, \ldots a_n\}$ is denoted:

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \ldots + a_n$$

Which is read "the sum of a_k from k = 1 to n."

The symbol Σ is simply an instruction to sum, or add up, the terms. The integer *k* is called the index of the sum, it tells you where to start the sum and where to end it. The expression

$\displaystyle\sum_{k=1}^{n} a_k$ is the instruction to add the terms a_k of the sequence $\{a_n\}$ from k = 1 through k = n.

Add all the numbers from 1 to 100.

(Story of Gauss)

1 + 2 + 3 + 4 + 5 + … + 99 + 100

100 + 99 + 98 + … + 2 + 1

101 X 100 = 10,100

10,100/2 = 5,050

Sum of n terms of an Arithmetic Sequence

Let { a_n} be an arithmetic sequence with first term a_1 and common difference d. The sum S_n of the first n terms of { a_n} is:

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + ... + a_n$$

$$= n\left(\frac{a_1 + a_n}{2}\right)$$

$$= \frac{n}{2}\left(2a_1 + (n-1)d\right)$$

See example 1 pg. 744

Sum of a Finite Geometric Sequence:

Let {a_n} be a finite geometric sequence with common ration r $\neq$ 1. Then the sum of the terms of the sequence is:

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + ... + a_n$$

$$= \frac{a_1(1-r^n)}{1-r}$$

See Example 2 pg. 745

Infinite Series:

An infinite series is an expression of the form

$$\sum_{k=1}^{\infty} a_k = a_1 + a_2 + a_3 + ... + a_n + ...$$

In some cases the sequence of **partial sums** approaches a finite limit S:

$$\lim_{n\to\infty} \sum_{k=1}^{n} a_k = \lim_{n\to\infty} (a_1 + a_2 + ... a_n) = S$$

In this case we say that the series **converges** to S, and it makes sense to define S as the **sum of the infinite series**.

$$\sum_{k=1}^{\infty} a_k = \lim_{n\to\infty} \sum_{k=1}^{n} a_k = S$$

If the limit of a partial sum does not exist, then the series **diverges** and has no sum.

See Example 3 pg. 747

Sum of an Infinite Geometric Series:

The geometric series $\sum_{k=1}^{\infty} a \bullet r^{k-1}$ converges if and only if $|r|<1$. If it does converge, the sum is

$$S = \frac{a}{1-r}.$$

See Example 4 pg. 748